

Exercise 15

Find the limit or show that it does not exist.

$$\lim_{x \rightarrow \infty} \frac{3x - 2}{2x + 1}$$

Solution

Multiply the numerator and denominator by the reciprocal of the highest power of x in the denominator.

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{3x - 2}{2x + 1} &= \lim_{x \rightarrow \infty} \frac{3x - 2}{2x + 1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{(3x - 2)\frac{1}{x}}{(2x + 1)\frac{1}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{3 - \frac{2}{x}}{2 + \frac{1}{x}} \\ &= \frac{\lim_{x \rightarrow \infty} \left(3 - \frac{2}{x}\right)}{\lim_{x \rightarrow \infty} \left(2 + \frac{1}{x}\right)} \\ &= \frac{\lim_{x \rightarrow \infty} 3 - \lim_{x \rightarrow \infty} \frac{2}{x}}{\lim_{x \rightarrow \infty} 2 + \lim_{x \rightarrow \infty} \frac{1}{x}} \\ &= \frac{3 - 0}{2 + 0} \\ &= \frac{3}{2} \end{aligned}$$